Register Allocation and Coalescing

- Introduction
- Abstraction and the Problem
- Algorithm
- Spilling
- Coalescing

Reading: ALSU 8.8.4
Motivation

• **Problem**
  – Allocation of variables (pseudo-registers) to hardware registers in a procedure

• **A very important optimization!**
  – Directly reduces running time
    • (memory access → register access)
  – Useful for other optimizations
    • e.g. CSE assumes old values are kept in registers.
Goals

• Find an allocation for all pseudo-registers, if possible.

• If there are not enough registers in the machine, choose registers to spill to memory
Register Assignment Example

A = ...
IF A goto L1

B = ...
  = A
D = ...
  = B + D

L1: C = ...
  = A
D = ...
  = C + D

• Find an assignment (no spilling) with only 2 registers
  – A and D in one register, B and C in another one

• What assumptions?
  – After assignment, no use of A & (and only one of B and C used)
An Abstraction for Allocation & Assignment

• **Intuitively**
  – Two pseudo-registers **interfere** if at some point in the program they cannot both occupy the same register.

• **Interference graph**: an **undirected** graph, where
  – nodes = pseudo-registers
  – there is an edge between two nodes if their corresponding pseudo-registers interfere

• **What is not represented**
  – Extent of the interference between uses of different variables
  – Where in the program is the interference

Interfere many times vs. once

E.g., cold path vs. hot path
Register Allocation and Coloring

• A graph is **n-colorable** if:
  – every node in the graph can be colored with one of the n colors such that two adjacent nodes do not have the same color.

• Assigning n register (without spilling) = Coloring with n colors
  – assign a node to a register (color) such that no two adjacent nodes are assigned same registers (colors)

• Is spilling necessary? = Is the graph n-colorable?

• To determine if a graph is n-colorable is **NP-complete, for n>2**
  – Too expensive
  – Heuristics
Algorithm

Step 1. Build an interference graph
   a. refining notion of a node
   b. finding the edges

Step 2. Coloring
   – use heuristics to try to find an n-coloring
     • Success:
       – colorable and we have an assignment
     • Failure:
       – graph not colorable, or
       – graph is colorable, but it is too expensive to color
Step 1a. Nodes in an Interference Graph

A = ...
IF A goto L1

B = ...
= A
D =
= B + D

L1: C = ...
= A
D =
= D + C

A = 2

Interference Graph

Should we add A-D edge?
No, since new def of A
Live Ranges and Merged Live Ranges

• Motivation: to create an interference graph that is easier to color
  – Eliminate interference in a variable’s “dead” zones.
  – Increase flexibility in allocation:
    • can allocate same variable to different registers
• A **live range** consists of a definition and all the points in a program in which that definition is live.
  – How to compute a live range?
• Two overlapping live ranges for the **same** variable must be merged
Example (Revisited)

Live Variables
Reaching Definitions

```
A = ... (A₁)
IF A goto L1

B = ... (B₁)
= A
D = B (D₂)

L1:
C = ... (C₁)
= A
D = ... (D₁)

A = 2 (A₂)
```

Live Variables
Reaching Definitions

```
{A} {A₁}
{A,B} {A₁,B₁}
{B} {A₁,B₁}
{D} {A₁,B₁,D₂}

{A} {A₁}
{A,C} {A₁,C₁}
{C} {A₁,C₁}
{D} {A₁,C₁,D₁}

{D} {A₁,B₁,C₁,D₁,D₂} {A,D} {A₂,B₁,C₁,D₁,D₂}

{A,D} {A₂,B₁,C₁,D₁,D₂}
{D} {A₂,B₁,C₁,D₁,D₂}

= A
ret D
```
Merging Live Ranges

• **Merging definitions into equivalence classes**
  – Start by putting each definition in a different equivalence class
  – Then, for each point in a program:
    • if (i) variable is live, and (ii) there are multiple reaching definitions for the variable, then:
      – merge the equivalence classes of all such definitions into one equivalence class
    • Sounds familiar?

• From now on, refer to **merged live ranges** simply as **live ranges**
  – merged live ranges are also known as “webs”
SSA Revisited: What Happens to $\Phi$ Functions

- Now we see why it is unnecessary to “implement” a $\Phi$ function
  - $\Phi$ functions and SSA variable renaming simply turn into merged live ranges

- When you encounter: $X_4 = \Phi(X_1, X_2, X_3)$
  - merge $X_1, X_2, X_3$, and $X_4$ into the same live range
  - delete the $\Phi$ function

- Now you have effectively converted back out of SSA form
Step 1b. Edges of Interference Graph

• **Intuitively:**
  – Two live ranges (necessarily of different variables) may *interfere* if they overlap at some point in the program.
  – Algorithm:
    • At each point in the program:
      – enter an edge for every pair of live ranges at that point.

• **An optimized definition & algorithm for edges:**
  – Algorithm:
    • check for interference only at the start of each live range
  – Faster
  – Better quality
Live Range Example 2

Because ranges overlap: Won’t assign A and B to same register (even though would have been ok: path sensitive vs. path insensitive analysis)
Step 2. Coloring

• Reminder: **coloring for n > 2 is NP-complete**

• **Observations:**
  – a node with degree $< n \Rightarrow$
    • can always color it successfully, given its neighbors’ colors
  – a node with degree $= n \Rightarrow$
    • can only color if at least two neighbors share same color
  – a node with degree $> n \Rightarrow$
    • maybe, not always

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{D}
\end{array}
\]
**Coloring Algorithm**

- **Algorithm:**
  - Iterate until stuck or done
    - Pick any node with degree < n
    - Remove the node and its edges from the graph
  - If done (no nodes left)
    - reverse process and add colors

- **Example (n = 3):**

  - **Note:** degree of a node may drop in iteration
  - Avoids making arbitrary decisions that make coloring fail
More details

- **Apply coloring heuristic**
  
  Build interference graph
  
  Iterate until there are no nodes left
  
  If there exists a node $v$ with less than $n$ neighbor
  
  push $v$ on register allocation stack
  
  else
  
  return *(coloring heuristics fail)*
  
  remove $v$ and its edges from graph

- **Assign registers**
  
  While stack is not empty
  
  Pop $v$ from stack
  
  Reinsert $v$ and its edges into the graph
  
  Assign $v$ a color that differs from all its neighbors
What Does Coloring Accomplish?

- **Done:**
  - colorable, also obtained an assignment

- **Stuck:**
  - colorable or not?

![Diagram](image-url)
Extending Coloring: Design Principles

• **A pseudo-register is**
  – Colored successfully: allocated a hardware register
  – Not colored: left in memory

• **Objective function**
  – Cost of an uncolored node:
    • proportional to number of uses/definitions (dynamically)
    • estimate by its loop nesting
  – Objective: minimize sum of cost of uncolored nodes

• **Heuristics**
  – Benefit of spilling a pseudo-register:
    • increases colorability of pseudo-registers it interferes with
    • can approximate by its degree in interference graph
  – Greedy heuristic
    • spill the pseudo-register with lowest cost-to-benefit ratio, whenever spilling is necessary
Spilling to Memory

- **CISC architectures**
  - can *operate on data in memory directly*
  - memory operations are *slower than register operations*

- **RISC architectures**
  - machine instructions can *only apply to registers*
  - **Use**
    - *must first load data from memory to a register before use*
  - **Definition**
    - *must first compute RHS in a register*
    - *store to memory afterwards*
  - Even if spilled to memory, needs a register at time of use/definition
Chaitin: Coloring and Spilling

• **Identify spilling**
  Build interference graph
  Iterate until there are no nodes left
  If there exists a node v with less than n neighbor
      place v on stack to register allocate
  else
      v = node with highest degree-to-cost ratio
      mark v as spilled
      remove v and its edges from graph

• **Spilling may require use of registers; change interference graph**
  While there is spilling
  rebuild interference graph and perform step above

• **Assign registers**
  While stack is not empty
  Remove v from stack
  Reinsert v and its edges into the graph
  Assign v a color that differs from all its neighbors
Spilling

• **What should we spill?**
  – Something that will eliminate a lot of interference edges
  – Something that is used infrequently
  – Maybe something that is live across a lot of calls?

• **One Heuristic:**
  – spill cheapest live range (aka “web”)
  – Cost = \[\frac{[(\text{# defs} \& \text{ uses}) \times 10^{\text{loop-nest-depth}}]}{\text{degree}}\]
Quality of Chaitin’s Algorithm

• Giving up too quickly

• \( N=2 \)

• An optimization: “Prioritize the coloring”
  – Still eliminate a node and its edges from graph
  – Do not commit to “spilling” just yet
  – Try to color again in assignment phase.
Splitting Live Ranges

• **Recall:** Split pseudo-registers into live ranges to create an interference graph that is easier to color
  – Eliminate interference in a variable’s “dead” zones.
  – Increase flexibility in allocation:
    • can allocate same variable to different registers
Insight

• Split a live range into smaller regions (by paying a small cost) to create an interference graph that is easier to color
  – Eliminate interference in a variable’s “nearly dead” zones.
    • Cost: Memory loads and stores
      – Load and store at boundaries of regions with no activity
    • # active live ranges at a program point can be > # registers

– Can allocate same variable to different registers
  • Cost: Register operations
    – a register copy between regions of different assignments
  • # active live ranges cannot be > # registers
Examples

Example 1:

```
FOR i = 0 TO 10
    FOR j = 0 TO 10000
        A = A + ...
        (does not use B)
    FOR j = 0 TO 10000
        B = B + ...
        (does not use A)
```

Example 2:

```
a =

b = c =
    a + b  a + c

b = c =
    b + c
```
Example 1

FOR \(i = 0\) TO 10

\[
\begin{align*}
\text{FOR } j &= 0 \text{ TO } 10000 \\
A &= A + \ldots \\
\text{(does not use } B) \\
\text{FOR } j &= 0 \text{ TO } 10000 \\
B &= B + \ldots \\
\text{(does not use } A) \\
\end{align*}
\]
Example 2

\[
\begin{align*}
    a &= \\
    b &= a + b \\
    c &= \\
    &= b + c \\
    \end{align*}
\]

\[
\begin{align*}
    a &= \\
    b &= a + b \\
    c &= a_1 + c \\
    &= b + c \\
    \end{align*}
\]

\[
\begin{align*}
    n &= 2 \\
    \end{align*}
\]

Can’t 2-color

Can 2-color
(“a” gets 2 regs)
Live Range Splitting

• When do we apply live range splitting?
• Which live range to split?
• Where should the live range be split?
• How to apply live-range splitting with coloring?
  – Advantage of coloring:
    • defers arbitrary assignment decisions until later
  – When coloring fails to proceed, may not need to split live range
    • degree of a node \( \geq n \) does not mean that the graph definitely is not colorable
  – Interference graph does not capture positions of a live range
One Algorithm

• **Observation**: spilling is absolutely necessary if
  – number of live ranges active at a program point > n

• **Apply live-range splitting before coloring**
  – Identify a point where number of live ranges > n
  – For each live range active around that point:
    • find the outermost “block construct” that does not access the variable
  – Choose a live range with the largest inactive region
  – Split the inactive region from the live range
Summary

• Problems:
  – Given n registers in a machine, is spilling avoided?
  – Find an assignment for all pseudo-registers, whenever possible.

• Solution:
  – Abstraction: an interference graph
    • nodes: live ranges
    • edges: presence of live range at time of definition
  – Register Allocation and Assignment problems
    • equivalent to n-colorability of interference graph
      \( \mathsf{NP}\text{-complete} \)
  – Heuristics to find an assignment for n colors
    • successful: colorable, and finds assignment
    • not successful: colorability unknown & no assignment
CSC D70: Compiler Optimization
Register Coalescing

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The content of this lecture is adapted from the lectures of Todd Mowry and Phillip Gibbons
Let’s Focus on Copy Instructions

- Optimizations that help optimize away copy instructions:
  - Copy Propagation
  - Dead Code Elimination

- Can all copy instructions be eliminated using this pair of optimizations?
Example Where Copy Propagation Fails

- Use of copy target has multiple (conflicting) reaching definitions

```plaintext
X = A + B;
Y = C;
Y = X;
Z = Y + 4;
```
Another Example Where the Copy Instruction Remains

- Copy target ($Y$) still live even after some successful copy propagations

- **Bottom line:**
  - copy instructions may still exist when we perform register allocation
Copy Instructions and Register Allocation

• What clever thing might the register allocator do for copy instructions?

  ![Copy Instructions Example]

  
  ...  
  \[ Y = X; \]  
  ...

• If we can assign both the source and target of the copy to the same register:
  – then we don’t need to perform the copy instruction at all!
  – the copy instruction can be removed from the code
    • even though the optimizer was unable to do this earlier

• One way to do this:
  – treat the copy source and target as the same node in the interference graph
    • then the coloring algorithm will naturally assign them to the same register
  – this is called “coalescing”
Simple Example: Without Coalescing

• Without coalescing, \( \mathbf{X} \) and \( \mathbf{Y} \) can end up in different registers
  – cannot eliminate the copy instruction

\[
\begin{align*}
\mathbf{X} &= \ldots; \\
\mathbf{A} &= 5; \\
\mathbf{Y} &= \mathbf{X}; \\
\mathbf{B} &= \mathbf{A} + 2; \\
\mathbf{Z} &= \mathbf{Y} + \mathbf{B}; \\
\text{return } \mathbf{Z};
\end{align*}
\]
Example Revisited: With Coalescing

- **With coalescing**, \(X\) and \(Y\) are now guaranteed to end up in the same register
  - the copy instruction can now be eliminated

- **Great! So should we go ahead and do this for every copy instruction?**
Should We Coalesce X and Y In This Case?

- It is legal to coalesce X and Y for a “Y = X” copy instruction iff:
  - initial definition of Y’s live range is this copy instruction, AND
  - the live ranges of X and Y do not interfere otherwise

- But just because it is legal doesn’t mean that it is a good idea...

\[
\begin{align*}
X &= A + B; \\
Y &= X; \\
X &= 2; \\
Z &= Y + X;
\end{align*}
\]

No! That would result in incorrect behavior if this branch is taken.
Why Coalescing May Be Undesirable

• What is the likely impact of coalescing \( X \) and \( Y \) on:
  – live range size(s)?
    • recall our discussion of live range splitting
  – colorability of the interference graph?
• Fundamentally, coalescing adds further constraints to the coloring problem
  – doesn’t make coloring easier; may make it more difficult
• If we coalesce in this case, we may:
  – save a copy instruction, BUT
  – cause significant spilling overhead if we can no longer color the graph
When to Coalesce

• Goal when coalescing is legal:
  – coalesce *unless* it would make a colorable graph *non-colorable*

• The bad news:
  – predicting colorability is tricky!
    • it depends on the shape of the graph
    • graph coloring is NP-hard

• **Example**: assuming 2 registers, should we coalesce **X** and **Y**?

![Diagram](image)
Representing Coalescing Candidates in the Interference Graph

- To decide whether to coalesce, we augment the interference graph
- Coalescing candidates are represented by a **new type of interference graph edge**:
  - dotted lines: coalescing candidates
    - *try* to assign vertices the **same color**
      - (unless that is problematic, in which case they can be given different colors)
  - solid lines: interference
    - vertices **must** be assigned **different colors**

```plaintext
X = ...;
A = 5;
Y = X;
B = A + 2;
Z = Y + B;
return Z;
```
How Do We Know When Coalescing Will Not Cause Spilling?

• **Key insight:**
  – Recall from the coloring algorithm:
    • we can always successfully N-color a node if its degree is < N

• To ensure that *coalescing does not cause spilling*:
  – check that the degree < N invariant is still locally preserved after coalescing
    • if so, then coalescing won’t cause the graph to become non-colorable
  – no need to inspect the entire interference graph, or do trial-and-error

• **Note:**
  – We do NOT need to determine whether the full graph is colorable or not
  – Just need to check that coalescing does not cause a colorable graph to become non-colorable
Simple and Safe Coalescing Algorithm

- We can safely coalesce nodes $X$ and $Y$ if $(|X| + |Y|) < N$
  - Note: $|X|$ = degree of node $X$ counting interference (not coalescing) edges

- **Example:**

  $(|X| + |Y|) = (1 + 2) = 3$

- Degree of coalesced node can be no larger than 3
  - if $N >= 4$, it would always be safe to coalesce these two nodes
    - this cannot cause new spilling that would not have occurred with the original graph
  - if $N < 4$, it is unclear

  *How can we (safely) be more aggressive than this?*
What About This Example?

• Assume $N = 3$
• Is it safe to coalesce $X$ and $Y$?

Notice: $X$ and $Y$ share a common (interference) neighbor: node $A$
  – hence the degree of the coalesced $X/Y$ node is actually 2 (not 3)
  – therefore coalescing $X$ and $Y$ is guaranteed to be safe when $N = 3$
• How can we adjust the algorithm to capture this?
Another Helpful Insight

• Colors are not assigned until nodes are popped off the stack
  – nodes with degree < N are pushed on the stack first
  – when a node is popped off the stack, we know that it can be colored
    • because the number of potentially conflicting neighbors must be < N

• Spilling only occurs if there is no node with degree < N to push on the stack

• Example: (N=2)
Another Helpful Insight

|X| = 5
|Y| = 5

2-colorable after coalescing $X$ and $Y$?
Building on This Insight

• When would coalescing cause the stack pushing (aka “simplification”) to get stuck?
  1. coalesced node must have a degree >= N
     • otherwise, it can be pushed on the stack, and we are not stuck
  2. AND it must have at least N neighbors that each have a degree >= N
     • otherwise, all neighbors with degree < N can be pushed before this node
       – reducing this node’s degree below N (and therefore we aren’t stuck)

• To coalesce more aggressively (and safely), let’s exploit this second requirement
  – which involves looking at the degree of a coalescing candidate’s neighbors
    • not just the degree of the coalescing candidates themselves
Briggs’s Algorithm

• Nodes X and Y can be coalesced if:
  – (number of neighbors of X/Y with degree \( \geq N \)) < N
• Works because:
  – all other neighbors can be pushed on the stack before this node,
  – and then its degree is < N, so then it can be pushed
  – Example: \( N = 2 \)
Briggs’s Algorithm

• Nodes $X$ and $Y$ can be coalesced if:
  – $(\text{number of neighbors of } X/Y \text{ with degree } \geq N) < N$

• More extreme example: $(N = 2)$
George’s Algorithm

Motivation:
• imagine that \textbf{X} has a very high degree, but \textbf{Y} has a much smaller degree
  – (perhaps because \textbf{X} has a large live range)

• With Briggs’s algorithm, we would inspect all neighbors both \textbf{X} and \textbf{Y}
  – but \textbf{X} has a lot of neighbors!
• Can we get away with just inspecting the neighbors of \textbf{Y}?
  – showing that coalescing makes coloring no worse than it was given \textbf{X}?

![Diagram](image_url)
George’s Algorithm

• Coalescing $X$ and $Y$ does no harm if:
  – foreach neighbor $T$ of $Y$, either:
    1. degree of $T$ is $<N$, or
    2. $T$ interferes with $X$

• Example: $(N=2)$

  similar to Briggs: $T$ will be pushed before $X/Y$
  hence no change compared with coloring $X$
Summary

- **Coalescing** can enable register allocation to eliminate copy instructions
  - if both source and target of copy can be allocated to the same register
- However, coalescing must be applied with care to avoid causing register spilling
- Augment the interference graph:
  - dotted lines for coalescing candidate edges
  - try to allocate to same register, unless this may cause spilling
- **Coalescing Algorithms**:
  - simply based upon degree of coalescing candidate nodes (X and Y)
  - Briggs’s algorithm
    - look at degree of neighboring nodes of X and Y
  - George’s algorithm
    - asymmetrical: look at neighbors of Y (degree and interference with X)